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5D action for longitudinal five branes on a pp-wave

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Abstract: String modes in a pp-wave background are generically massive, and the worldvolume description of the branes is to be given by 'massive' gauge theories. In this paper, we present a five dimensional super Yang-Mills action with the Kähler-Chern-Simons term plus the Myers term as a low energy worldvolume description of the longitudinal five branes in a maximally supersymmetric pp-wave background. We derive the action from the M-theory matrix model on the pp-wave. We utilize the previously found 4/32 BPS solution of rotating five branes with stacks of membranes, but, to obtain the static configuration, we reformulate the matrix model in a rotating coordinate system which provides the inertial frame for the branes. Expanding the matrix model around the solution, we first obtain a non-commutative field theory action naturally equipped with the full sixteen dynamical supersymmetries. In the commutative limit, we show only four supersymmetries survive, resulting in a novel five dimensional "$\mathcal{N} = 1/2$" theory.

Keywords: M(atrix) Theories, Brane Dynamics in Gauge Theories, Supersymmetric Effective Theories.

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Contents

1. Introduction

Recently [1], Berenstein, Maldacena and Nastase (BMN) proposed a novel matrix model which describes M-theory in the maximally supersymmetric pp-wave background of the eleven dimensional supergravity [2, 3, 4],

$$ds^2 = -2dx^+dx^- - \left[\left(\frac{\mu}{6}\right)^2 (x_1^2 + \cdots + x_6^2) + \left(\frac{\mu}{3}\right)^2 (x_7^2 + x_8^2 + x_9^2)\right] dx^+dx^+ + \sum_{A=1}^{9} dx^A dx^A \,, \tag{1.1}$$

$$F_{+789} = \mu \,, \tag{1.2}$$

where μ becomes the characteristic mass parameter of the matrix model. The resulting matrix model corresponds to a mass deformation of the BFSS matrix model [5]–[8], still maintaining the full supersymmetries, sixteen dynamical and sixteen kinematical. The BMN matrix model was also shown to agree with the matrix regularization [9, 10] of the supermembrane on the pp-wave geometry [11].

Due to the mass parameter, the BMN matrix model captures many interesting novel properties. The supersymmetry transformations have the explicit time dependence so that the supercharges do not commute with the hamiltonian. As a result, the bosons and fermions have different masses. The bosonic mass terms lift up the flat directions completely, and the perturbative expansion is possible by powers of the dimensionless

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parameter, $(\mu l_p^2/R)^{-1}$, where l_p is the eleven dimensional Planck length and R is the radius of the null compactification [11, 12]. Classical vacua are given by fuzzy spheres sitting at the origin stretching over the $7, 8, 9$ directions.

In [13] (see also [14]), the supersymmetry algebra of the BMN matrix model was identified as the special unitary Lie superalgebra of which the complexification corresponds to A(1|3), and the classification of the quantum BPS multiplets was carried out as its atypical representations. Soon after, in [15], the classical counterparts of the quantum BPS states were studied. Namely, all the BPS equations which correspond to the quantum BPS states preserving some fraction of the dynamical supersymmetry were obtained. The results show that there are essentially one unique set of 2/16 BPS equations, three inequivalent sets of 4/16 BPS equations, and three inequivalent sets of 8/16 BPS equations only, in addition to the 16/16 static fuzzy sphere. The solutions include the known ones, rotating longitudinal five branes with stacks of D2 branes in them [16], rotating ellipsoidal branes, rotating or static hyperboloids [17], rotating fuzzy torus [18], and also new ones such as the rotating fuzzy spheres or D0 branes in various directions with different supersymmetries, a static fuzzy sphere on a hyperboloid, a mixture of rotating two hyperboloids and a fuzzy sphere [15].

Especially, among them the solution describing rotating longitudinal five branes with stacks of D2 branes is of particular interest in the present paper. From the classification of the BPS equations it appears that the solution is the unique 'flat' longitudinal five brane solution which preserves only the dynamical supersymmetries. The configuration satisfies 'the su(2) singlet 4/16 BPS equations' so that it preserves four dynamical supersymmetries only. This contrasts to the BFSS matrix model or $\mu = 0$ case where the longitudinal five brane with stacks of D2 branes preserves half of thirty two supersymmetries. More detailed comparison is given later.

It is also worth to note that there are supersymmetric configurations which preserve only certain nontrivial combinations of the dynamical and kinematical supersymmetries. They include a transverse membrane and a longitudinal five brane [16]. Since the kinematical supercharges and the dynamical supercharges in the BMN matrix model have different quantum numbers for the hamiltonian, such configurations do not correspond to the energy eigenstates but rather superpositions.

One characteristic feature of the string theory in a pp-wave background is that the string modes are generically massive [19]–[22],

$$E_n = \sqrt{\mu^2 + \frac{n^2}{(\alpha' p_+)^2}}\,. \tag{1.3}$$

Therefore, in the $\alpha' \to 0$ limit, the worldvolume descriptions of the branes are to be given by 'massive' gauge theories.[1]

In this paper, we present a five dimensional super Yang-Mills action with the Kähler-Chern-Simons term plus the Myers term as a low energy worldvolume description of the longitudinal five branes in a maximally supersymmetric pp-wave background. We derive the action in the M-theory matrix model setup. We utilize the known BPS solution of

[1] An attempt to build such field theories was taken in [23].

rotating five branes with stacks of transverse membranes or D2 branes, but, to obtain the static configuration, we reformulate the BMN matrix model in a rotating coordinate system which provides the inertial frame for the branes. The modified matrix model naturally admits flat and static longitudinal five branes with stacks of D2 branes in them which preserve four dynamical supersymmetries. We first expand the modified matrix model around the solution, and obtain a non-commutative field theory naturally equipped with the full sixteen supersymmetries. Taking the commutative limit and letting the D2 branes disappear, we finally get the worldvolume action for the longitudinal five . We show only four supersymmetries survive, resulting in a novel five dimensional "$\mathcal{N}=1/2$" theory.

The organization of the present paper is as follows. In section 2, we first reformulate the BMN matrix model by introducing a new coordinate system. In this setup, we identify the BPS equations for the supersymmetric configurations which preserve four dynamical supersymmetries, and as a special solution we find flat and static longitudinal five branes with stacks of D2 branes in them. Expanding the matrix model around the solution we derive a non-commutative five dimensional $\mathrm{U}(N)$ super Yang-Mills action with the Kähler-Chern-Simons term plus the Myers term equipped with the full sixteen dynamical supersymmetries. In section 3 we take the commutative limit to obtain the worldvolume action for the longitudinal five branes on the pp-wave. The D2 branes are now gone and the resulting commutative action has only four supersymmetries. We study the supersymmetry algebra and identify the central and R-symmetry charges. We consider the BPS configurations which preserve all the four supersymmetries and write the corresponding BPS equations. We also discuss the energy spectra of the bosons and fermions, and show that the five dimensional $\mathcal{N}=1/2$ model contains three supermultiplets. Finally, in section 4, we conclude with the summary. The appendix contains some useful formulae.

2. M-theory matrix model on a fully supersymmetric pp-wave

2.1 BMN matrix model in the rotating coordinate system

The original BMN matrix model or the M-theory matrix model on a fully supersymmetric pp-wave background admits the rotating flat longitudinal five branes as a BPS solution preserving four supersymmetries [16, 15]. For the purpose of the present paper, we choose the comoving or inertial coordinate system such that the longitudinal five brane solution becomes static. Explicitly we replace the first four coordinates, x_1, x_2, x_3, x_4, by the $\mathrm{SO}(2)\times\mathrm{SO}(2)$ rotating ones,

$$x_1 \to \cos(\mu x^+/6)x_1 + \sin(\mu x^+/6)x_2\,, \qquad x_2 \to \cos(\mu x^+/6)x_2 - \sin(\mu x^+/6)x_1\,, \tag{2.1}$$

$$x_3 \to \cos(\mu x^+/6)x_3 + \sin(\mu x^+/6)x_4\,, \qquad x_4 \to \cos(\mu x^+/6)x_4 - \sin(\mu x^+/6)x_3\,, \tag{2.2}$$

so that the metric of the eleven dimensional pp-wave background (1.2) is, in the new coordinate system, of the form

$$\begin{aligned} ds^2 \,=\, & -2dx^+dx^- - \frac{\mu}{3}\left(x_1dx_2 - x_2dx_1 + x_3dx_4 - x_4dx_3\right)dx^+ - \\ & - \left[\left(\frac{\mu}{6}\right)^2 (x_5^2+x_6^2) + \left(\frac{\mu}{3}\right)^2 (x_7^2+x_8^2+x_9^2)\right] dx^+dx^+ + \sum_{A=1}^{9} dx^A dx^A\,. \end{aligned} \tag{2.3}$$

The corresponding M-theory matrix model on this background is then obtained from the original BMN matrix model by taking the above time dependent SO(2)×SO(2) rotation. With $t \equiv x^{+}$, the transformation of the bosons is essentially the same as above (cf. [24, 22]),

$$X_1 \rightarrow \cos(\mu t/6)X_1 + \sin(\mu t/6)X_2\,, \qquad \text{etc.} \tag{2.4}$$

while that of the fermions reads, from the standard Lorentz transformation rule,

$$\Psi \rightarrow e^{\frac{\mu}{12}(\Gamma^{12}+\Gamma^{34})t}\Psi\,. \tag{2.5}$$

The modified, but nevertheless equivalent, M-theory matrix model on a fully supersymmetric pp-wave background spells with a mass parameter, μ,

$$\mathcal{S} = \frac{l_p^6}{R^3}\int dt\mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \tag{2.6}$$

$$\begin{aligned}
\mathcal{L}_0 &= \mathrm{Tr}\left(\frac{1}{2}D_tX^AD_tX_A + \frac{1}{4}\left[X^A,X^B\right]^2 + i\frac{1}{2}\Psi^\dagger D_t\Psi - \frac{1}{2}\Psi^\dagger\Gamma^A\left[X_A,\Psi\right]\right)\,,\\
\mathcal{L}_1 &= \mathrm{Tr}\left[-\frac{1}{6}J^{ij}X_iD_tX_j - i\frac{1}{3}\epsilon^{rst}X_rX_sX_t + i\frac{1}{24}\Psi^\dagger\left(\Gamma^{12}+\Gamma^{34}+3\Gamma^{789}\right)\Psi\right]\,,\\
\mathcal{L}_2 &= -\frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{1}{6}\right)^2\left(X_5^2+X_6^2\right) + \left(\frac{1}{3}\right)^2\left(X_7^2+X_8^2+X_9^2\right)\right]\,,
\end{aligned} \tag{2.7}$$

where $i,j = 1,2,3,4$, $r,s,t = 7,8,9$, $A,B = 1,2,\dots,9$ and J^{ij} is a skew-symmetric constant two form of which the non-vanishing components are $J^{12} = J^{34} = 1$ only, up to the anti-symmetric property. In the present paper, we adopt generic Euclidean nine dimensional gamma matrices, $\Gamma^A = (\Gamma^A)^\dagger$, $\Gamma^{12\cdots9} = 1$. Namely we do not adopt the usual real and symmetric Majorana representation. Accordingly there exits a nontrivial 16×16 charge conjugation matrix, C,

$$\left(\Gamma^A\right)^T = \left(\Gamma^A\right)^* = C^{-1}\Gamma^AC\,, \qquad C = C^T = (C^\dagger)^{-1}\,. \tag{2.8}$$

The spinors, Ψ, satisfy the Majorana condition leaving eight independent complex components,

$$\Psi = C\Psi^*\,. \tag{2.9}$$

The covariant derivatives are in our convention, $D_t\mathcal{O} = \frac{d}{dt}\mathcal{O} - i[A_0,\mathcal{O}]$ so that X and A_0 are of the mass dimension one, while Ψ has the mass dimension 3/2.

Compared to the original BMN matrix model, the quadratic mass terms for the bosonic first four coordinates are absent. Instead, there appear terms linear in μ as well as the velocities. Consequently the linearly realized isometry group is broken as

$$\mathrm{SO}(6)\times\mathrm{SO}(3) \rightarrow \mathrm{SU}(2)\times\mathrm{SO}(2)\times\mathrm{SO}(3)\,, \tag{2.10}$$

which is the price we pay in order to get the static flat longitudinal five brane configurations we discuss shortly.

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The supersymmetry transformations are

$$\delta A_0 = i\Psi^\dagger \mathcal{E}(t)\,, \qquad \delta X^A = i\Psi^\dagger\Gamma^A\mathcal{E}(t)\,, \tag{2.11}$$

$$\begin{aligned}\delta\Psi =& \Big[D_tX^A\Gamma_A - i\frac{1}{2}\left[X^A,X^B\right]\Gamma_{AB} + \frac{\mu}{6}\big(X^5\Gamma_5+X^6\Gamma_6-2X^7\Gamma_7-2X^8\Gamma_8-2X^9\Gamma_9\big)\Gamma^{789} + \\ &+\frac{\mu}{6}\left(X^1\Gamma_1+X^2\Gamma_2\right)\left(\Gamma^{789}-\Gamma^{12}\right)+\frac{\mu}{6}\left(X^3\Gamma_3+X^4\Gamma_4\right)\left(\Gamma^{789}-\Gamma^{34}\right)\Big]\mathcal{E}(t)\,,\end{aligned} \tag{2.12}$$

where

$$\mathcal{E}(t) = e^{\frac{\mu}{12}(-\Gamma^{12}-\Gamma^{34}+\Gamma^{789})t}\mathcal{E}\,, \qquad \mathcal{E} = C\mathcal{E}^*\,, \tag{2.13}$$

and $\mathcal{E}$ is a sixteen component constant spinor.

In addition there is the kinematical supersymmetry,

$$\delta A_0 = \delta X^A = 0\,, \qquad \delta\Psi = e^{-\frac{\mu}{12}(\Gamma^{12}+\Gamma^{34}+3\Gamma^{789})t}\mathcal{E}'\,, \qquad \mathcal{E}' = C\mathcal{E}'^*\,. \tag{2.14}$$

2.2 Static longitudinal five branes preserving four supersymmetries

In general, the Killing spinors in the supersymmetry transformations form a kernel space. Analyzing 'the projection matrix' to the kernel, one can obtain in a systematic way all the possible sets of the BPS equations of various unbroken supersymmetry fractions [25, 15]. In order to obtain the static longitudinal five brane configuration, it is convenient to consider the following 4/16 projection matrix for the Killing spinors [15],

$$\Omega = \frac{1}{4}\left(1-\Gamma^{1234}-\Gamma^{3456}-\Gamma^{5612}\right)\,, \tag{2.15}$$

which satisfies

$$\Omega^\dagger = \Omega\,, \qquad C\Omega^*C^{-1} = \Omega\,, \qquad \Omega^2 = \Omega\,, \qquad \mathrm{tr}\,\Omega = 4\,. \tag{2.16}$$

Now replacing the Killing spinor, $\mathcal{E}$, in (2.12) by the projection matrix, rewriting the expression in terms of the totally anti-symmetric products of gamma matrices and requiring each coefficient to vanish one can obtain the following BPS equations preserving four supersymmetries,

$$\begin{aligned} D_tZ_1 = D_tZ_2 = D_tX_r &= 0\,, \qquad D_tZ_3 + i\frac{\mu}{6}Z_3 = 0\,,\\ [X_r,X_s] - i\frac{\mu}{3}\epsilon_{rst}X^t &= 0\,, \qquad [X_r,X_A] = 0,\ A = 1,2,\dots,6\,,\\ [Z_1,Z_2] &= 0\,, \qquad [Z_1,\bar{Z}_1]+[Z_2,\bar{Z}_2]+[Z_3,\bar{Z}_3] = 0\,,\\ [Z_2,Z_3] &= 0\,, \qquad [Z_3,Z_1] = 0\,, \end{aligned} \tag{2.17}$$

where we complexify the coordinates as $Z_1 = X_1+iX_2$, $Z_2 = X_3+iX_4$, $Z_3 = X_5+iX_6$, and set $\bar{Z}_1 = (Z_1)^\dagger$ etc. Note that the BPS equations themselves imply the Gauss constraint. Rotating back to the original coordinates, $Z_1, Z_2 \to e^{i\mu t/6}Z_1, e^{i\mu t/6}Z_2$, this set of BPS equations is identical to the su(2) singlet BPS equations preserving four supersymmetries found in [15].

Generic finite matrix solutions describe the fuzzy sphere or the giant graviton expanding in the $7,8,9$ directions and rotating on the $(5,6)$ plane with the frequency, $\mu/6$, since the last four equations imply that Z_1, Z_2, Z_3 are simultaneously diagonalizable. On the other hand, for the infinite matrix solutions, by setting $X_r = Z_3 = A_0 = 0$, one can obtain the static flat longitudinal five branes [16],

$$[X^1, X^2] + [X^3, X^4] = 0\,, \quad [X^1, X^3] + [X^4, X^2] = 0\,, \quad [X^1, X^4] + [X^2, X^3] = 0\,. \tag{2.18}$$

In the present paper, we consider the longitudinal five branes with stacks of D2 branes in them [26] as solutions,

$$X^i = i\hat{\partial}_i\,, \qquad i = 1, 2, 3, 4\,. \tag{2.19}$$

Here $\hat{\partial}_i$'s are related to the coordinates of a four dimensional non-commutative space,

$$x^i = i\theta^{ij}\hat{\partial}_j\,, \tag{2.20}$$

such that

$$\left[x^i, x^j\right] = i\theta^{ij}\,, \qquad \left[\hat{\partial}_i, \hat{\partial}_j\right] = i\theta^{-1}_{ij}\,, \qquad \left[\hat{\partial}_i, x^j\right] = \delta_i{}^j\,. \tag{2.21}$$

In order to satisfy the BPS condition (2.18), the noncommutative parameter must satisfy the anti-self-duality,

$$\theta^{ij} + \frac{1}{2}\epsilon^{ijkl}\theta^{kl} = 0 \qquad \Longleftrightarrow \qquad \theta^{-1}_{ij} + \frac{1}{2}\epsilon_{ijkl}\theta^{-1}_{kl} = 0\,. \tag{2.22}$$

The relation (2.21) defines a pair of non-commutative planes, and hence two sets of the harmonic oscillators. The most general irreducible representation is then specified by the superselection rule which is the number of the ground states that we denote by N. Thus, the Hilbert space, $\mathcal{H}$, on which the infinite matrices act decomposes as a direct product of two harmonic oscillator Hilbert spaces, $H_{\rm h.o.}$ and an N dimensional vector space, V_N,

$$\mathcal{H} = H_{\rm h.o.} \oplus H_{\rm h.o.} \oplus V_N\,. \tag{2.23}$$

Explicitly as in [27], using the bra and ket notation one can regroup the states in the Hilbert space as

$$|n_1, n_2, s\rangle\,, \qquad n_1, n_2 = 0, 1, \ldots, \infty\,, \qquad s = 1, 2, \ldots, N\,, \tag{2.24}$$

so that the two creation operators are

$$\sum_{n_1,n_2,s} \sqrt{n_1+1}|n_1+1, n_2, s\rangle\langle n_1, n_2, s|\,, \qquad \sum_{n_1,n_2,s} \sqrt{n_2+1}|n_1, n_2+1, s\rangle\langle n_1, n_2, s|\,. \tag{2.25}$$

In terms of branes, this represents N parallel longitudinal five branes on top of each other with stacks of D2 branes in them, which preserve four supersymmetries.

It is worth to note that in the ordinary BFSS matrix model or the $\mu = 0$ case, the same longitudinal five brane configuration, (2.19), (2.21), (2.22), preserves sixteen supersymmetries out of thirty two. They are eight of the dynamical supersymmetries with the projection matrix, $\frac{1}{4}(1 - \Gamma^{1234})$, and eight linear combinations of the kinematical and dynamical supersymmetries, since the remaining dynamical supersymmetry transformations of the fermions are canceled by the kinematical supersymmetry transformations. Furthermore, it is possible to relax the anti-self-duality condition (2.22). In that case, the longitudinal five brane configuration preserves sixteen linear combinations of the kinematical and dynamical supersymmetries. However, in the case of $\mu \neq 0$, the mixing between the kinematical and dynamical supersymmetries is not allowed because of the different time dependence in (2.13) and (2.14). In summary, the flat longitudinal five branes are 4/32 supersymmetric in the pp-wave background, while 16/32 supersymmetric in the flat background.

2.3 Non-commutative 5D super Yang-Mills-Kähler-Chern-Simons-Myers action

In this subsection, we expand our M-theory matrix model around the supersymmetric N parallel longitudinal five brane solution above, and derive a five dimensional super Yang-Mills action coupled to the Kähler-Chern-Simons term plus the Myers term.

Introducing the gauge fields as the longitudinal fluctuations around the five brane solution, we write the bosonic variables as

$$X_i = i\hat{\partial}_i + A_i\,, \qquad i = 1,2,3,4\,, X_a = \Phi_a\,, \qquad a = 5,6,7,8,9\,. \tag{2.26}$$

Consequently

$$D_t X_i = F_{0i}\,, \qquad [X_i, X_j] = i(F_{ij} - \theta^{-1}_{ij})\,, \qquad [X_i, \Phi] = iD_i\Phi\,, \qquad [X_i, \Psi] = iD_i\Psi\,, \tag{2.27}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, $\mu, \nu = 0,1,2,3,4$ and the derivative along the non-commutative coordinate of a function is from (2.21), $\partial_i\Phi = [\hat{\partial}_i, \Phi]$. The fields have the standard gauge transformation properties,

$$A_\mu \to UA_\mu U^\dagger + iU\partial_\mu U^\dagger\,, \qquad \Phi \to U\Phi U^\dagger\,. \tag{2.28}$$

To write the matrix model (2.6) in terms of the gauge fields we first note

$$J^{ij}\,\mathrm{Tr}(X_i D_t X_j) = -\frac{1}{2}\epsilon^{\lambda\mu\nu ij}\,\mathrm{Tr}\Big(A_\lambda\partial_\mu A_\nu - i\frac{2}{3}A_\lambda A_\mu A_\nu\Big)J_{ij} + J^{ij}\theta^{-1}_{ij}\,\mathrm{Tr}\,A_0 + \frac{d}{dt}\,\mathrm{Tr}\Big(iJ^{ij}\hat{\partial}_i A_j\Big)\,, \tag{2.29}$$

where $\epsilon^{\lambda\mu\nu ij}$ is the totally anti-symmetric five form tensor with $\epsilon^{01234} = 1$. Now the crucial observation to make is that the second term linear in A_0 on the right hand side vanishes due to the anti-self-duality of the non-commutative parameter, $J^{ij}\theta^{-1}_{ij} = 0$. Therefore the right hand side is identified as the Kähler-Chern-Simons term [28] up to the total derivative with J_{ij} being the Kähler form in the non-commutative flat four dimensional space. Since the left hand side is manifestly gauge invariant, there will be no quantization rule for the coefficient of the Kähler-Chern-Simons term, contrary to the case in the Chern-Simons theory on a non-commutative plane [29].

Now using the fact that the trace over the Hilbert space, $\mathcal{H}$, can decompose into the integration over the non-commutative four dimensional space and the trace over the

"U(N)" indices,[2]

$$\mathrm{Tr}\,\mathcal{O}(x) = \frac{1}{(2\pi\theta)^2}\int dx^4\,\mathrm{tr}_N\,\mathcal{O}(x)\,, \tag{2.30}$$

our M-theory matrix model (2.6) in the five brane background becomes, discarding the total derivative terms and the mass of the five brane background, a non-commutative five dimensional super Yang-Mills action coupled to the Kähler-Chern-Simons term [28] plus the Myers term [30],

$$\mathcal{S} = \frac{1}{g_{\mathrm{YM}}^2}\int dx^5\ \mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \qquad g_{\mathrm{YM}}^2 = \frac{(2\pi\theta)^2 R^3}{l_p^6} = \frac{(2\pi\theta)^2 g_s}{l_s^3}\,, \tag{2.31}$$

$$\begin{aligned}
\mathcal{L}_0 &= \mathrm{tr}_N\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\Phi_a D^\mu\Phi_a + \frac{1}{4}[\Phi_a,\Phi_b]^2 - i\frac{1}{2}\Psi^\dagger\Gamma^\mu D_\mu\Psi - \frac{1}{2}\Psi^\dagger\Gamma^a[\Phi_a,\Psi]\right],\\
\mathcal{L}_1 &= \mathrm{tr}_N\left[\frac{1}{12}\epsilon^{\lambda\mu\nu ij}\,\mathrm{Tr}\left(A_\lambda\partial_\mu A_\nu - i\frac{2}{3}A_\lambda A_\mu A_\nu\right)J_{ij} - i\frac{1}{3}\epsilon^{rst}\Phi_r\Phi_s\Phi_t + i\frac{1}{48}\Psi^\dagger(\Gamma^{ij}J_{ij} + 6\Gamma^{789})\Psi\right],\\
\mathcal{L}_2 &= -\frac{1}{2}\,\mathrm{tr}_N\left[\left(\frac{1}{6}\right)^2(\Phi_5^2 + \Phi_6^2) + \left(\frac{1}{3}\right)^2(\Phi_7^2 + \Phi_8^2 + \Phi_9^2)\right],
\end{aligned} \tag{2.32}$$

where[3] $i = 1,2,3,4$, $a = 5,6,7,8,9$, $r = 7,8,9$, $\Gamma^0 = -1$, and our choice of the metric for the five dimensional minkowskian spacetime is $\eta = \mathrm{diag}(-++++)$. Any product is to be understood as the non-commutative star product.

The supersymmetry transformations are from (2.12)

$$\delta A_\mu = i\Psi^\dagger\Gamma_\mu\mathcal{E}(t)\,, \qquad \delta\Phi_a = i\Psi^\dagger\Gamma_a\mathcal{E}(t)\,, \tag{2.33}$$

$$\begin{aligned}
\delta\Psi =& \left[\frac{1}{2}F_{\mu\nu}\tilde{\Gamma}^\mu\Gamma^\nu + D_\mu\Phi_a\tilde{\Gamma}^\mu\Gamma^a - i\frac{1}{2}[\Phi_a,\Phi_b]\Gamma^{ab} - \tfrac{\mu}{12}\left(\Phi_a\Gamma^a\Gamma^{789} + 3\Gamma^{789}\Phi_a\Gamma^a\right)\right]\mathcal{E}(t) +\\
&+\left[\begin{array}{l}\frac{1}{2}\theta_{ij}^{-1}\Gamma^{ij} + \frac{\mu}{6}\left((\theta_{1i}^{-1}x^i + A_1)\Gamma^1 + (\theta_{2i}^{-1}x^i + A_2)\Gamma^2\right)(\Gamma^{789} - \Gamma^{12})\\ \frac{\mu}{6}\left((\theta_{3i}^{-1}x^i + A_3)\Gamma^3 + (\theta_{4i}^{-1}x^i + A_4)\Gamma^4\right)(\Gamma^{789} - \Gamma^{34})\end{array}\right]\mathcal{E}(t)\,,
\end{aligned} \tag{2.34}$$

where

$$\mathcal{E}(t) = e^{\frac{\mu}{12}(-\Gamma^{12} - \Gamma^{34} + \Gamma^{789})t}\mathcal{E}\,, \qquad \mathcal{E} = C\mathcal{E}^*\,. \tag{2.35}$$

Thus the full supersymmetry remains unbroken for this reformulation, which is no surprise as the non-commutative five dimensional action (2.31) is merely a particular manifestation of the background independent M-theory matrix model.

In the next section by taking the commutative limit, $\theta^{ij} \to 0$ while keeping g_{YM} fixed, we obtain the worldvolume action for the longitudinal five branes on the pp-wave without the stacks of the D2 branes, as their charge densities become

$$(l_p^6/R^3)[X^i, X^j] = g_{\mathrm{YM}}^{-2}\mathcal{O}(\theta) \to 0\,, \qquad (l_p^6/R^3)\epsilon_{0ijkl}X^iX^jX^kX^l = g_{\mathrm{YM}}^{-2} \times \mathrm{const}\,. \tag{2.36}$$

In particular we will see that the dynamical supersymmetry reduces from sixteen to four.[4]

[2]Here we set $\theta^2 = \mathrm{pfaffian}(\theta^{ij})$.

[3]Ten dimensional gamma matrices are in our convention, $\left(\begin{smallmatrix}0 & \tilde{\Gamma}^M\\ \Gamma^M & 0\end{smallmatrix}\right)$, $\tilde{\Gamma}^M = \Gamma_M$, $M = 0,1,2,\ldots,9$.

[4]Note that in field theories, contrary to the one dimensional matrix model, the kinematical supersymmetry is not physical at the quantum level, since the relevant supercharge would diverge with the space volume factor.

3. Worldvolume action for the longitudinal five branes on a pp-wave

3.1 Commutative five dimensional $\mathcal{N}=1/2$ worldvolume action

Taking the commutative limit,[5] $\theta^{ij}\rightarrow 0$, while keeping $g_{\rm YM}$ fixed, we first observe that the supersymmetry transformation of the fermions (2.34) becomes singular. To remedy the problem one should impose the following constraint on the Killing spinor,

$$\Gamma^{12}\mathcal{E}=\Gamma^{34}\mathcal{E}=\Gamma^{789}\mathcal{E}\,, \tag{3.1}$$

which also implies, with the anti-self-duality, $\theta^{-1}_{ij}\Gamma^{ij}\mathcal{E}=0$. The constraint is in fact equivalent to

$$\Omega\mathcal{E}=\mathcal{E}\,, \tag{3.2}$$

where Ω is the 4/16 projection matrix given in (2.15). Hence the unbroken supersymmetry of the longitudinal five branes reappear precisely as the supersymmetry of the worldvolume theory. In the commutative limit where the star product is replaced by the ordinary product, the action is of the same form as (2.31), (2.32), namely five dimensional super Yang-Mills-Kähler-Chern-Simons-Myers action with four supersymmetries. The supersymmetry transformations reduce to[6]

$$\begin{aligned}
\delta A_\mu &= i\Psi^\dagger\Gamma_\mu\mathcal{E}(t)\,, \qquad \delta\Phi_a=i\Psi^\dagger\Gamma_a\mathcal{E}(t)\,, \\
\delta\Psi &= \left[\frac{1}{2}F_{\mu\nu}\tilde{\Gamma}^\mu\Gamma^\nu+D_\mu\Phi_a\tilde{\Gamma}^\mu\Gamma^a-i\frac{1}{2}[\Phi_a,\Phi_b]\Gamma^{ab}-\frac{\mu}{12}\left(\Phi_a\Gamma^a\Gamma^{789}+3\Gamma^{789}\Phi_a\Gamma^a\right)\right]\mathcal{E}(t)\,,
\end{aligned} \tag{3.3}$$

where $a=5,6,7,8,9$ and

$$\mathcal{E}(t)=e^{-\frac{\mu}{12}\Gamma^{789}t}\mathcal{E}\,,\qquad \mathcal{E}=C\mathcal{E}^*\,,\qquad \Omega\mathcal{E}=\mathcal{E}\,. \tag{3.4}$$

As the five dimensional Lorentz symmetry is explicitly broken, the supersymmetry can be half of the "minimal" one, or "$\mathcal{N}=1/2$".

At this point, it is interesting to compare with the ordinary BFSS matrix model or the $\mu=0$ case. In that case, the only singular piece in the $\theta^{ij}\rightarrow 0$ limit of the supersymmetry transformation is $-\frac{1}{2}\theta^{-1}_{ij}\Gamma^{ij}\mathcal{E}$. Unlike the $\mu\neq 0$ case, this singularity can be removed by the kinematical supersymmetry transformations, as both the dynamical and kinematical supersymmetry transformations do not have the explicit time dependency when $\mu=0$. Thus in the $\mu=0$ case the full dynamical supersymmetry remains unbroken in the commutative limit. Nevertheless, both in the $\mu=0$ and $\mu\neq 0$ cases, the commutative worldvolume actions are equipped with the same number of supersymmetries the longitudinal five branes preserve, i.e. 16 for $\mu=0$ and 4 for $\mu\neq 0$.

From (3.2) it follows $\Gamma^{56}\mathcal{E}=\Gamma^{789}\mathcal{E}$ in addition to (3.1). Thus, if we redefine the fermions and two of the Higgs, using the time dependent SO(2) rotation,

$$\Psi\rightarrow e^{-\frac{\mu}{12}\Gamma^{56}t}\Psi\,,\quad \Phi_5\rightarrow\cos(\mu t/6)\Phi_5-\sin(\mu t/6)\Phi_6\,,\quad \Phi_6\rightarrow\cos(\mu t/6)\Phi_6+\sin(\mu t/6)\Phi_5\,, \tag{3.5}$$

[5] Due to the anti-self-duality, there is essentially only one parameter to take the limit.

[6] Direct manipulation in the commutative setup indeed shows that the above supersymmetry transformations subject to the constraint (3.4) leave the action invariant.

the explicit time dependency in the supersymmetry transformations will disappear.[7] In terms of the new variables, our $\mathcal{N}=1/2$ super Yang-Mills-Kähler-Chern-Simons-Myers action for the description of the longitudinal five branes on the pp-wave becomes

$$
\begin{aligned}
\mathcal{S} &= \frac{1}{g_{\rm YM}^2}\int dx^5 \mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \qquad g_{\rm YM} = \sqrt{R}\,, && (3.6)\\
\mathcal{L}_0 &= {\rm tr}_N\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\Phi_a D^\mu\Phi_a + \frac{1}{4}[\Phi_a,\Phi_b]^2 - i\frac{1}{2}\Psi^\dagger\Gamma^\mu D_\mu\Psi - \frac{1}{2}\Psi^\dagger\Gamma^a[\Phi_a,\Psi]\Big]\,,\\
\mathcal{L}_1 &= {\rm tr}_N\Big[\frac{1}{12}\epsilon^{\lambda\mu\nu ij}\,{\rm Tr}(A_\lambda\partial_\mu A_\nu - i\frac{2}{3}A_\lambda A_\mu A_\nu)J_{ij} + \frac{1}{6}\epsilon^{pq}\Phi_p D_0\Phi_q - i\frac{1}{3}\epsilon^{rst}\Phi_r\Phi_s\Phi_t\\
&\quad + i\frac{1}{24}\Psi^\dagger(\Gamma^{12}+\Gamma^{34}-\Gamma^{56}+3\Gamma^{789})\Psi\Big]\,,\\
\mathcal{L}_2 &= -\frac{1}{2}\,{\rm tr}_N\Big[(\tfrac{1}{3})^2(\Phi_7^2+\Phi_8^2+\Phi_9^2)\Big]\,, && (3.7)
\end{aligned}
$$

where $i=1,2,3,4$, $a=5,6,7,8,9$, $p=5,6$, $r=7,8,9$, and $\epsilon^{01234}=\epsilon^{56}=\epsilon^{789}=1$.

The supersymmetry transformations are

$$
\begin{aligned}
\delta A_\mu &= i\Psi^\dagger\Gamma_\mu\mathcal{E}\,, \qquad \delta\Phi_a = i\Psi^\dagger\Gamma_a\mathcal{E}\,,\\
\delta\Psi &= \Big[\frac{1}{2}F_{\mu\nu}\tilde{\Gamma}^\mu\Gamma^\nu + D_\mu\Phi_a\tilde{\Gamma}^\mu\Gamma^a - i\frac{1}{2}[\Phi_a,\Phi_b]\Gamma^{ab} + \frac{\mu}{3}\left(\Phi_p\Gamma^p - \Phi_r\Gamma^r\right)\Gamma^{789}\Big]\mathcal{E}\,, && (3.8)
\end{aligned}
$$

where $\mathcal{E}$ is a time independent constant spinor subject to $\mathcal{E}=C\mathcal{E}^*$ and $\Omega\mathcal{E}=\mathcal{E}$. Note that now the supersymmetry transformations do not have the explicit time dependency, which implies that the supercharges commute with the hamiltonian.

For the later reference, we give the equations of motion,

$$
\begin{aligned}
D_\nu F^\nu{}_0 + i[\Phi_a, D_0\Phi_a] + \frac{1}{2}\{\Psi^{\dagger\alpha},\Psi_\alpha\} - \frac{\mu}{3}(F_{12}+F_{34}+i[\Phi_5,\Phi_6]) &= 0\,,\\
D_\nu F^\nu{}_i + i[\Phi_a, D_i\Phi_a] + \frac{1}{2}\{\Psi^{\dagger\alpha},(\Gamma_i\Psi)_\alpha\} + \frac{\mu}{3}J_{ij}F_{j0} &= 0\,,\\
D_\mu D^\mu\Phi_p - [\Phi_a,[\Phi_a,\Phi_p]] + \frac{1}{2}\{\Psi^{\dagger\alpha},(\Gamma_p\Psi)_\alpha\} + \frac{\mu}{3}\epsilon_{pq}D_0\Phi_q &= 0\,,\\
D_\mu D^\mu\Phi_r - [\Phi_a,[\Phi_a,\Phi_r]] + \frac{1}{2}\{\Psi^{\dagger\alpha},(\Gamma_r\Psi)_\alpha\} - i\mu\epsilon_{rst}\Phi_s\Phi_t - (\mu 3)^2\Phi_r &= 0\,,\\
\Gamma^\mu D_\mu\Psi - i\Gamma^a[\Phi_a,\Psi] - \frac{\mu}{12}(\Gamma^{12}+\Gamma^{34}-\Gamma^{56}+3\Gamma^{789})\Psi &= 0\,. && (3.9)
\end{aligned}
$$

3.2 Supersymmetry algebra

The Noether charge of the supersymmetry can be written in terms of the supercharge and the supersymmetry parameter as

$$
i\int dx^4\,{\rm tr}_N\left(\Psi^\dagger\delta\Psi\right) = iQ^\dagger\mathcal{E} = -i\mathcal{E}^\dagger Q\,. \tag{3.10}
$$

The supercharge is explicitly of the form, with $a=5,6,7,8,9$,

$$
Q = \Omega\int dx^4\,{\rm tr}_N\Big[-\frac{1}{2}F_{\mu\nu}\Gamma^\mu\tilde{\Gamma}^\nu + D_\mu\Phi_a\Gamma^a\tilde{\Gamma}^\mu + i\frac{1}{2}[\Phi_a,\Phi_b]\Gamma^{ab} + \frac{\mu}{3}\Phi_a\Gamma^a\Gamma^{789}\Big]\Psi\,, \tag{3.11}
$$

[7] Note that the direction of the rotation is opposite to (2.4), (2.5).

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and satisfy

$$Q = C(Q^{\dagger})^{T}\,, \qquad Q = \Omega Q\,. \tag{3.12}$$

The supersymmetry algebra of the five dimensional $\mathcal{N} = 1/2$ worldvolume theory is found to be, after some tedious manipulation, (cf. [31])

$$\begin{aligned}
[H, Q] &= 0\,, & (3.13)\\
[M_{56}, Q] &= i\frac{1}{2}\Gamma_{56}Q\,, \qquad [M_{rs}, Q] = i\frac{1}{2}\Gamma_{rs}Q\,, &\\
[M_r, M_s] &= i\epsilon_{rst}M_t\,, \qquad M_r = \frac{1}{2}\epsilon_{rst}M_{st}\,, & (3.14)\\
\{Q, Q^{\dagger}\} &= 2\Omega\left[H - \mathcal{R} - \frac{\mu}{3}M_{56} + \Gamma^{r}\left(\mathcal{R}_r + \frac{\mu}{3}M_r\right) + \Gamma^{135r}\mathcal{A}_r + \Gamma^{246r}\mathcal{B}_r\right]\Omega\,. & (3.15)
\end{aligned}$$

Here H is the hamiltonian of which the bosonic part reads

$$H = \int dx^4\, \mathrm{tr}_N \left[\frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}D_0\Phi_a^2 + \frac{1}{2}D_i\Phi_a^2 - \frac{1}{4}\left[\Phi_a, \Phi_b\right]^2 + i\frac{\mu}{3}\epsilon^{rst}\Phi_r\Phi_s\Phi_t + \frac{1}{2}\left(\frac{\mu}{3}\right)^2\Phi_r^2\right], \tag{3.16}$$

M_{56}, M_{rs} are so(2), so(3) R-symmetry generators,

$$\begin{aligned}
M_{56} &= \int dx^4\, \mathrm{tr}_N \left[\epsilon^{pq}D_0\Phi_p\Phi_q - \frac{\mu}{6}\left(\Phi_5^2 + \Phi_6^2\right) - i\frac{1}{4}\Psi^{\dagger}\Gamma_{56}\Psi\right],\\
M_{rs} &= \int dx^4\, \mathrm{tr}_N \left[D_0\Phi_r\Phi_s - D_0\Phi_s\Phi_r - i\frac{1}{4}\Psi^{\dagger}\Gamma_{rs}\Psi\right], \qquad (3.17)
\end{aligned}$$

and $\mathcal{R}$, $\mathcal{R}_r$, $\mathcal{A}_r$, $\mathcal{B}_r$ are real central charges given by the boundary terms,

$$\begin{aligned}
\mathcal{R} &= \frac{1}{2}\int dx^4\partial_i\, \mathrm{tr}_N \left[J^{ij}\epsilon^{pq}\Phi_p D_j\Phi_q - \epsilon^{0ijkl}(A_j\partial_k A_l - i\frac{2}{3}A_jA_kA_l)\right],\\
\mathcal{R}_r &= \frac{1}{2}\epsilon_{rst}\int dx^4\partial_i\, \mathrm{tr}_N \left[J^{ij}D_j\Phi_s\Phi_t\right],\\
\mathcal{A}_r &= \int dx^4\partial_i\, \mathrm{tr}_N \left[h^{ij}\Phi_r(D_j\Phi_5 - J_{jk}D_k\Phi_6)\right],\\
\mathcal{B}_r &= -\int dx^4\partial_i\, \mathrm{tr}_N \left[h^{ij}\Phi_r(D_j\Phi_6 + J_{jk}D_k\Phi_5)\right], \qquad (3.18)
\end{aligned}$$

where we set $h^{31} = h^{24} = -h^{13} = -h^{42} = 1$ and others zero. Note that $\mathcal{R}$ contains the Chern-Pontryagin density, $F \wedge F$, which counts the number of D0 branes dissolved in the longitudinal five branes. For other central charges, we do not have clear interpretations yet in terms of the extended objects in the string theory.

The numbers of degrees in the left and right hand sides of (3.15) match as

$$10 = 1 + 3 + 3 + 3\,, \tag{3.19}$$

as Ω, $\Omega\Gamma^{r}\Omega$, $\Omega\Gamma^{135r}\Omega$, $\Omega\Gamma^{246r}\Omega$ are the only allowed independent gamma matrix products to appear on the right.

It is interesting to note that the spatial translation and the isometry of the Kähler form, SU(2), are not part of the $\mathcal{N} = 1/2$ supersymmetry algebra, though they are not broken. After all, $\mathcal{N}=1/2$ supersymmetry is too small to capture all the symmetries in the model. Compared to the supersymmetry algebra of the BMN matrix model [1, 16, 13, 32], the coefficient of M_{56} appearing in the anti-commutator of the supercharges is doubled from $\mu/6$ to $\mu/3$. This reflects our redefinition of Φ_5, Φ_6 by the rotating ones (3.5).

From the positive definity, we have the following energy bound,

$$H \geq \mathcal{R} + \frac{\mu}{3}M_{56} + \left|(\hat{e}_1)_r\left(\mathcal{R}_r + \frac{\mu}{3}M_r\right)\right| + |(\hat{e}_2)_r\mathcal{A}_r| + |(\hat{e}_3)_r\,\mathcal{B}_r|\ , \tag{3.20}$$

where $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ form an arbitrary orthonormal real basis for the "7, 8, 9" space so that $(\hat{e}_1)_r\Gamma^r$, $(\hat{e}_2)_r\Gamma^{135r}$, $(\hat{e}_3)_r\Gamma^{246r}$ can be simultaneously diagonalized with the eigenvalues, ±1.

3.3 BPS equations for the fully supersymmetric configurations

In this subsection we consider the BPS configurations which preserve all the four supersymmetries. In the conventional supersymmetric models, such fully supersymmetric configurations would be vacua, but in the present case, the novel structure of the supersymmetry algebra allows nontrivial fully supersymmetric BPS configurations. They have the energy saturation,

$$H = \mathcal{R} + \frac{\mu}{3}M_{56}\,, \tag{3.21}$$

while other central and R-symmetry charges vanish, $\mathcal{R}_r = \mathcal{A}_r = \mathcal{B}_r = M_r = 0$.

The corresponding BPS equations can be obtained either by writing $H - \mathcal{R} - \frac{\mu}{3}M_{56}$ as a sum of squares or from the supersymmetry transformation of the fermions (A.1),

$$\begin{aligned}
F_{0\mu} = D_0\Phi_r &= 0\,, &\quad D_0\Phi_p - \frac{\mu}{3}\epsilon_{pq}\Phi_q &= 0\,,\\
[\Phi_r,\Phi_s] - i\frac{\mu}{3}\epsilon_{rst}\Phi_t &= 0\,, &\quad D_i\Phi_r = [\Phi_5,\Phi_r] = [\Phi_6,\Phi_r] &= 0\,,\\
F_{13}+F_{42} &= 0\,, &\quad F_{14}+F_{23} &= 0\,,\\
F_{12}+F_{34} - i[\Phi_5,\Phi_6] &= 0\,, &\quad D_i\Phi_5 - J_{ij}D_j\Phi_6 &= 0\,,
\end{aligned} \tag{3.22}$$

where $i=1,2,3,4$, $p=5,6$, $r=7,8,9$, and the BPS equations themselves satisfy the Gauss constraint. In particular, the so(2) R-symmetry charge becomes

$$M_{56} = \frac{\mu}{6}\int dx^4\,\mathrm{tr}_N\left(\Phi_5^2+\Phi_6^2\right). \tag{3.23}$$

These BPS equations are the same as the BPS equations, (2.17), in the original M-theory matrix model up to the field redefinition (3.5). After all, the flat longitudinal five brane is just a particular solution of the latter and the BPS equations above in the worldvolume theory can be interpreted as the constraint for the D0 branes dissolved in the five branes which still preserve the four supersymmetries.

The last four BPS equations are essentially identical to the BPS equations in Euclidean six dimensional super Yang-Mills theory [25]. When all the Higgs are turned off, the BPS equations reduce to the well known anti-self-dual equations for the field strength, $F_{ij}+\frac{1}{2}\epsilon_{ijkl}F_{kl}=0$, for which the ADHM construction provides the general solutions. On the other hand, just like in the BMN matrix model, the classical supersymmetric vacua are given by the constant fuzzy spheres,

$$[\Phi_r,\Phi_s] = i\tfrac{\mu}{3}\epsilon_{rst}\Phi_t\,,\qquad \Phi_5=\Phi_6=F_{\mu\nu}=0\,. \tag{3.24}$$

energy spectra	Ψ	A_μ	Φ_5, Φ_6	Φ_7, Φ_8, Φ_9
$E_{\mathbf{k}} = \sqrt{(\frac{\mu}{3})^2 + \mathbf{k}^2}$	4	1	0	3
$E^{+}_{\mathbf{k}} = \sqrt{(\frac{\mu}{6})^2 + \mathbf{k}^2} + \frac{\lvert\mu\rvert}{6}$	2	1	1	0
$E^{-}_{\mathbf{k}} = \sqrt{(\frac{\mu}{6})^2 + \mathbf{k}^2} - \frac{\lvert\mu\rvert}{6}$	2	1	1	0

Table 1: Energy spectra and the numbers of bosons and fermions.

3.4 Energy spectra and supermultiplets

In order to clarify the supermultiplet contents, we investigate the energy spectra of the bosons and fermions. This can be done by considering the equations of motion (3.9) for the free or U(1) case.

For the fermions, if we consider the plane wave solution, $\Psi(x) = \psi_k e^{ik\cdot x}$, the equation of motion becomes

$$\left[i\Gamma^\mu k_\mu - \frac{\mu}{12}\left(\Gamma^{12} + \Gamma^{34} - \Gamma^{56} + 3\Gamma^{789}\right)\right]\psi_k = 0\,. \tag{3.25}$$

To admit a nontrivial solution it is necessary to impose[8]

$$\det\left[i\Gamma^\mu k_\mu - \frac{\mu}{12}\left(\Gamma^{12} + \Gamma^{34} - \Gamma^{56} + 3\Gamma^{789}\right)\right] = \left(k^2 + (\tfrac{\mu}{3})^2\right)^2\left(k^2 + \frac{\mu}{3}k_0\right)\left(k^2 - \frac{\mu}{3}k_0\right) = 0\,. \tag{3.26}$$

Thus, the fermions have the following three energy spectra,

$$E_{\mathbf{k}} = \sqrt{\left(\frac{\mu}{3}\right)^2 + \mathbf{k}^2}\,, \qquad E^{+}_{\mathbf{k}} = \sqrt{\left(\frac{\mu}{6}\right)^2 + \mathbf{k}^2} + \frac{|\mu|}{6}\,, \qquad E^{-}_{\mathbf{k}} = \sqrt{\left(\frac{\mu}{6}\right)^2 + \mathbf{k}^2} - \frac{|\mu|}{6}\,. \tag{3.27}$$

For each spectrum there are four, two and two fermionic modes, respectively.

Similarly one can obtain the energy spectra for the bosons. The gauge fields consist of three independent modes having the above three energy spectra, $E_{\mathbf{k}}, E^{+}_{\mathbf{k}}, E^{-}_{\mathbf{k}}$, respectively. The Higgs fields, Φ_5, Φ_6, decompose into two modes which have the energy spectra, $E^{+}_{\mathbf{k}}, E^{-}_{\mathbf{k}}$, while the other three Higgs fields, Φ_r, have only one spectrum, $E_{\mathbf{k}}$.

In fact, the $\pm|\mu|/6$ factors in $E^{\pm}_{\mathbf{k}}$ are the reminiscent of the coordinate transformations using the SO(2) rotations (2.4), (3.5). They coincide with the frequencies of the rotations.

The energy spectra of the bosons and fermions are summarized in table 1.

Clearly each line forms a separate supermultiplet. Note that it is the coefficient of the Fourier mode, the creation or annihilation operator, not the c-number part, $e^{ik\cdot x}$, that transforms under the adjoint action of the supercharges on the canonically quantized fields.

4. Conclusion

We have obtained a five dimensional U(N) $\mathcal{N} = 1/2$ super Yang-Mills action with the Kähler-Chern-Simons term and the Myers term as a low energy worldvolume description of the longitudinal five branes in a maximally supersymmetric pp-wave background.

[8] In the manipulation of the determinant we used the explicit representation of the gamma matrices given in the appendix.

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We derived the action utilizing the known rotating longitudinal five brane solution preserving four supersymmetries in the BMN matrix model. Adopting the inertial or co-moving frame, we reformulate the matrix model in a new coordinate system which involves the replacement of some bosonic mass terms by the "Chern-Simons" term and the modification of the fermion's mass term. In this setup the 'flat' and 'static' longitudinal five brane solution was identified and shown to preserve four dynamical supersymmetries. Expanding the modified matrix model around the solution, we first obtained a non-commutative field theory naturally equipped with the full sixteen dynamical supersymmetries. In the commutative limit, we showed only four supersymmetries survive, resulting in the $\mathcal{N} = 1/2$ model.

In the original BMN matrix model which is written in the maximally symmetric coordinate system, the longitudinal five branes should rotate in order to preserve the supersymmetries. The Kähler structure in the worldvolume action is inherited from the rotating directions of the longitudinal five branes. In this sense, due to the presence of the Kähler form, the five dimensional Lorentz symmetry is spontaneously broken. This accounts the emergence of the half of the "minimal" supersymmetry in five dimensions.

We wrote the supersymmetry algebra explicitly, identifying all the possible central charges. Thanks to the novel structure of the algebra, the $\mathcal{N} = 1/2$ model admits the BPS configurations which preserve all the four supersymmetries. In particular, when all the Higgs fields are turned off, they reduce to the ordinary anti-self-dual equations for the field strength, while the classical supersymmetric vacua are given by the constant fuzzy spheres.

We obtained the novel energy spectra of the bosons and fermions in the worldvolume action. The results show that the model contains three different supermultiplets embedded in a nontrivial way.

The resulting worldvolume action possesses four supersymmetries, which is natural as we started with the five brane configuration preserving four dynamical supersymmetries in the matrix model. According to the classification of the BPS equations in the BMN matrix model [15], it appears that there is no flat longitudinal five brane configuration which preserves other than four dynamical supersymmetries. This is certainly true within the matrix formulation setup of the M-theory. However, recently it was shown that the mass deformation of the DLCQ matrix model for the longitudinal five branes [33] is possible, while keeping eight dynamical supersymmetries [34]. This might suggest that, just like the transverse five branes, more supersymmetric longitudinal five branes may exist in the M-theory on the pp-wave, which the matrix model can not capture.

Contrary to the D-brane worldvolume actions in the flat background, the conventional dimensional reduction of the present five dimensional $\mathcal{N} = 1/2$ action would not correspond to the T-duality of string theory due to the nontrivial pp-wave geometry. The worldvolume actions for other branes on the pp-wave should be obtained case by case.

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A. Useful formulae

A useful identity to derive the BPS equations preserving all the four supersymmetries in the worldvolume action is

$$\begin{aligned}
&\left[\frac{1}{2}F_{\mu\nu}\tilde{\Gamma}^{\mu}\Gamma^{\nu}+D_{\mu}\Phi_{a}\tilde{\Gamma}^{\mu}\Gamma^{a}-i\frac{1}{2}\left[\Phi_{a},\Phi_{b}\right]\Gamma^{ab}+\frac{\mu}{3}\left(\Phi_{p}\Gamma^{p}-\Phi_{r}\Gamma^{r}\right)\Gamma^{789}\right]\Omega=\\
&=F_{0i}\Gamma^{i}+D_{0}\Phi_{r}\Gamma^{r}+D_{i}\Phi_{r}\Gamma^{ir}+\left(D_{0}\Phi_{p}-\frac{\mu}{3}\epsilon_{pq}\Phi_{q}\right)\Gamma^{p}-i\frac{1}{2}\left(\left[\Phi_{r},\Phi_{s}\right]-i\frac{\mu}{3}\epsilon_{rst}\Phi_{t}\right)\Gamma^{rs}-\\
&\quad-i\left[\Phi_{p},\Phi_{r}\right]\Gamma^{pr}+\left(D_{i}\Phi_{5}-J_{ij}D_{j}\Phi_{6}\right)\Gamma^{i5}+\left(F_{13}+F_{42}\right)\Gamma^{13}+\left(F_{14}+F_{23}\right)\Gamma^{14}+\\
&\quad+\left(F_{12}+F_{34}-i\left[\Phi_{5},\Phi_{6}\right]\right)\Gamma^{12}\,,
\end{aligned}\tag{A.1}$$

where $i=1,2,3,4$, $p=5,6$, $r=7,8,9$.

Evaluating the anti-commutator of the supercharges to derive the supersymmetry algebra (3.15), one needs the following Fierz identities for the nine dimensional gamma matrices, $(\Gamma^{A})_{\alpha}{}^{\beta}$, $\alpha,\beta=1,2,\ldots,16$,

$$\begin{aligned}
&\delta^{\alpha}{}_{\gamma}\delta^{\beta}{}_{\delta}-\delta^{\alpha}{}_{\delta}\delta^{\beta}{}_{\gamma}=\frac{1}{16}(C^{-1}\Gamma^{AB})^{\alpha\beta}(\Gamma_{AB}C)_{\gamma\delta}+\frac{1}{48}(C^{-1}\Gamma^{ABC})^{\alpha\beta}(\Gamma_{ABC}C)_{\gamma\delta}\,, &\text{(A.2)}\\
&\left(\Gamma^{AB}\right)_{\alpha}{}^{\gamma}(C^{-1}\Gamma_{B})^{\beta\delta}+(C^{-1}\Gamma^{AB})^{\beta\delta}(\Gamma_{B})_{\alpha}{}^{\gamma}+(\gamma\leftrightarrow\delta)=2(\Gamma^{A})_{\alpha}{}^{\beta}C^{-1\gamma\delta}-2\delta_{\alpha}{}^{\beta}(C^{-1}\Gamma^{A})^{\gamma\delta}\,.
\end{aligned}$$

In the manipulation of the determinant (3.26), we used the following representation of the "ten" dimensional gamma matrices,

$$\Gamma^{0}=-1\otimes 1\,,\qquad \Gamma^{m}=1\otimes\gamma^{m}\,,\qquad \Gamma^{r}=\sigma^{r-6}\otimes\gamma^{(7)}\,,\tag{A.3}$$

where $m=1,2,\ldots,6$, $r=7,8,9$, $\sigma^{1},\sigma^{2},\sigma^{3}$ are the usual Pauli matrices, and γ^{m}'s are the six dimensional gamma matrices,

$$\gamma^{m}=\begin{pmatrix}0&\rho^{m}\\(\rho^{m})^{\dagger}&0\end{pmatrix},\qquad \gamma^{(7)}=i\gamma^{4}\gamma^{5}\cdots\gamma^{9}=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\tag{A.4}$$

with the anti-symmetric 4×4 matrices [35],

$$\begin{aligned}
&\rho^{1}=\begin{pmatrix}i\epsilon&0\\0&-i\epsilon^{-1}\end{pmatrix},\qquad \rho^{2}=\begin{pmatrix}\epsilon&0\\0&\epsilon^{-1}\end{pmatrix},\qquad \rho^{3}=\begin{pmatrix}0&i\sigma^{3}\\-i(\sigma^{3})^{T}&0\end{pmatrix},\\
&\rho^{4}=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\qquad \rho^{5}=\begin{pmatrix}0&i\sigma^{1}\\-i(\sigma^{1})^{T}&0\end{pmatrix},\qquad \rho^{6}=\begin{pmatrix}0&i\sigma^{2}\\-i(\sigma^{2})^{T}&0\end{pmatrix}.
\end{aligned}\tag{A.5}$$

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